Name:__________________________

SM3 12.2: Basic Trig Proof

Vocab: Conjecture: An unproven statement, believed to be true

Deductive Reasoning: The process of reaching a logical certainty

QED (*quod erat demonstrandum*): Latin for "which had to be demonstrated"; we use this to signify that we have completed our proof

Direct Mathematical Proof: Using theorems and axioms to demonstrate that an equation is always true

Direct Mathematical Proof is not the only type of proof but it is the only type we'll be mastering in this course. The goal is always the same: cause the left side of the equation to be identical to the right side of the equation by using the math theorems you've learned. Each step taken is to be documented so that the reader of the proof can follow the logical reasoning of the proof.

The target audience for our proving will be Secondary Math 3 students. This means that you can refer to theorems by the names that you'd expect a Secondary Math 3 student to be familiar with without having to also prove those theorems. If we were proving equations to a 5-year-old, we'd have a lot more work to do; if we were proving equations to an advanced calculus student, some of our steps might be able to be combined or implied.

Example: Prove $\frac{(7+2)^2}{3} = 27$

Strategically speaking, this will not be very challenging because both sides of the equation have the same number of terms. We should be able to use familiar order of operations theorems to do the job.

Statements	**Reasons**
$\frac{(7+2)^2}{3} = 27$	Given
$\frac{(9)^2}{3} = 27$	Addition
$\frac{81}{3} = 27$	Multiplication
$27 = 27$	Division

QED

Example: Prove $\sin^3(\theta)\cot(\theta)\sec^3(\theta) = \tan(\theta)$

Both sides of the equation have the same number of terms, so we just need to simplify the left side. Let's start by using trigonometric definitions to change all trig functions into $\sin(\theta)$ and $\cos(\theta)$.

Statements	**Reasons**
$\sin^3(\theta)\cot^2(\theta)\sec^3(\theta) = \tan(\theta)$	Given
$\sin^3(\theta)\dfrac{\cos^2(\theta)}{\sin^2(\theta)}\sec^3(\theta) = \tan(\theta)$	Definition of $\cot(\theta)$
$\sin^3(\theta)\dfrac{\cos^2(\theta)}{\sin^2(\theta)}\dfrac{1}{\cos^3(\theta)} = \tan(\theta)$	Definition of $\sec(\theta)$
$\dfrac{\sin^3(\theta)\cos^2(\theta)}{\sin^2(\theta)\cos^3(\theta)} = \tan(\theta)$	Multiplication
$\dfrac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$	Division
$\tan(\theta) = \tan(\theta)$	Definition of $\tan(\theta)$
QED	

Example: Prove $\tan(\theta) + \csc(\theta) = \frac{\sin^2(\theta)+\cos(\theta)}{\sin(\theta)\cos(\theta)}$

The left side has two terms; the right side has one messy term. Let's start by combining the two terms on the left by finding a common denominator.

Statements	**Reasons**
$\tan(\theta) + \csc(\theta) = \dfrac{\sin^2(\theta) + \cos(\theta)}{\sin(\theta)\cos(\theta)}$	Given
$\dfrac{\sin(\theta)}{\cos(\theta)} + \csc(\theta) = \dfrac{\sin^2(\theta) + \cos(\theta)}{\sin(\theta)\cos(\theta)}$	Definition of $\tan(\theta)$
$\dfrac{\sin(\theta)}{\cos(\theta)} + \dfrac{1}{\sin(\theta)} = \dfrac{\sin^2(\theta) + \cos(\theta)}{\sin(\theta)\cos(\theta)}$	Definition of $\csc(\theta)$
$\dfrac{\sin(\theta)\sin(\theta)}{\sin(\theta)\cos(\theta)} + \dfrac{\cos(\theta)}{\sin(\theta)\cos(\theta)} = \dfrac{\sin^2(\theta) + \cos(\theta)}{\sin(\theta)\cos(\theta)}$	Multiplication
$\dfrac{\sin(\theta)\sin(\theta) + \cos(\theta)}{\sin(\theta)\cos(\theta)} = \dfrac{\sin^2(\theta) + \cos(\theta)}{\sin(\theta)\cos(\theta)}$	Addition
$\dfrac{\sin^2(\theta) + \cos(\theta)}{\sin(\theta)\cos(\theta)} = \dfrac{\sin^2(\theta) + \cos(\theta)}{\sin(\theta)\cos(\theta)}$	Multiplication

QED

Use two columns to prove each identity.

1) $4(3-5)^2 = 16$

Statements	**Reasons**
$4(3-5)^2 = 16$	Given

2) $(6-9)(2-8) = 18$

3) $\sec(\theta)\cos(\theta) = 1$

4) $\frac{1}{\sin(\theta)} = \csc(\theta)$

5) $3\tan(\theta) + 4\tan(\theta) = 7\tan(\theta)$

6) $-\tan(\theta)\csc(\theta) = -\sec(\theta)$

7) $\sin(\theta)\cot(\theta) = \cos(\theta)$

8) $\cos(\theta)\tan(\theta) = \sin(\theta)$

9) $\sin(\theta)\cot(\theta)\tan(\theta) = \sin(\theta)$

10) $\tan^2(\theta)\cos^4(\theta) = \sin^2(\theta)\cos^2(\theta)$

11) $(\sin(\theta)+1)(\sin(\theta)-1)=\sin^2(\theta)-1$

12) $(\cos(\theta)+1)(\cos(\theta)+4)=\cos^2(\theta)+5\cos(\theta)+4$

13) $\tan^2(\theta)-9=(\tan(\theta)-3)(\tan(\theta)+3)$

14) $(\csc(\theta)-2)(\csc(\theta)+2)+4=\frac{1}{\sin^2(\theta)}$

15) $\frac{\sin^2(\theta)+10\sin(\theta)+24}{\sin^2(\theta)-16}=\frac{\sin(\theta)+6}{\sin(\theta)-4}$

16) $\frac{2\cot^2(\theta)-5\cot(\theta)-3}{\cot^2(\theta)-10\cot(\theta)+21}=\frac{2\cot(\theta)+1}{\cot(\theta)-7}$

17) $\frac{\cos^2(\theta)+\sin^2(\theta)}{\sin^2(\theta)}=\cot^2(\theta)+1$

18) $\frac{\csc(\theta)+3\sec(\theta)}{5\csc(\theta)}=\frac{1}{5}+\frac{3}{5}\tan(\theta)$

19) $\tan(\theta)+\sec(\theta)=\frac{\sin(\theta)+1}{\cos(\theta)}$

20) $\sec(\theta)+\sin(\theta)=\frac{1+\sin(\theta)\cos(\theta)}{\cos(\theta)}$